2101001102010001
EXAMINATION FEBRUARY-MARCH 2024
MASTER OF ARTS EXTERNAL PART-2
MATHEMATICS PAPER-501
DIFFERENTIAL GEOMETRY & LINEAR ALGEBRA LEVEL 1

[**Time: As Per Schedule]** **[Max. Marks: 100]**

Instructions:
1. Fill up strictly the following details on your answer book
a. Name of the Examination: **MASTER OF ARTS EXTERNAL PART-2**
b. Name of the Subject: **MATHEMATICS PAPER-501 DIFFERENTIAL GEOMETRY & LINEAR ALGEBRA LEVEL 1**
c. Subject Code No: **2101001102010001**

2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.

Seat No:

Student's Signature

Q.1 A) Find the involutes of the circular helix **6**
x = a cos θ, y = a sin θ, z = aθ tan α.

B) Prove that in order that the principal normal of a curve be binomial's of another, the relation a $(k^2 + \tau^2)$ = bk must hold, where a and b are constants. **7**

C) Obtain the equations of a circular helix **7**
r = (a cos u, a sin u, bu), -∞ < u < ∞.

OR

A) Show that a necessary and sufficient condition for a curve to be helix is that the ratio of the curvature and torsion is constant. **6**

B) Find the basic unit vectors t, n, b of the curve r = (u, u^2, u^3) at the point u = 1. Also find the equations of tangent, the principal normal and binormal at this point. **7**

C) Establish briefly the Serret-Frenet formulae at a point of a space curve. **7**

Q.2 A) Explain the terms (i) characteristic of the surface F (x, y, z, a) = 0 for the parameter a, (ii) Envelope of the family of surface F (x, y, z, a) = 0, (iii) Edge of regression. **6**

B) Show that the sum of the squares of the intercepts on the co-ordinates axes made by the tangent plane to the surface $x^{2/3} + y^{2/3} + z^{2/3} = a^{2/3}$ is constant. **7**

C) Find the necessary and sufficient condition for the surface z = f(x, y) to be developable. Also prove that surface xy = $(z - c)^2$ is developable. **7**

OR

A) Find the equation to the developable surface which has $x = 6t, y = 3t^2$, Z=2t^3 for edge of regression. **7**

B) Prove that the characteristic of a family of surface of one parameter are tangent to the edge of regression. **6**

C) Define Tangent plane. Find the equation for the tangent plane to the surface z =$x^2 + y^2$ at the point (1,-1,2). **7**

Q.3 A) Define normal curvature. State and prove Meusnier's theorem. **7**

B) Calculate the first and second order magnitudes for the coincoid x = u cos v, y = u sin v, z = f(v) with u, v as parameters. **7**

C) Find the radii of curvature and torsion of the helix x= a cos u, y= a sin u, z = au tan α. **6**

OR

A) Find the normal curvature of a normal section of the right helicoid X= u cos v, y= u sin v, z=cv. **6**

B) Prove that for any curve **7**

$$[t', t'', t'''] = [r'', r''', r''''] = k^3 (k\tau' - k'\tau) = k^5 \frac{d}{ds}\left(\frac{\tau}{k}\right) \text{ and}$$

$$[b', b'', b'''] = \tau^3(k'\tau - \tau' k) = \tau^5 \frac{d}{ds}\left(\frac{k}{\tau}\right)$$

C) Define First and second order fundamental form. Give geometrical interpretation of second fundamental form. **7**

Q.4 A) State and prove Rank-Nullity theorem. **7**

B) Find a basis for the range space of the transformation given by the matrix **6**

$$A=\begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & -3 & -2 & -5 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

C) For $A=\begin{bmatrix} 3 & -3 & 1 \\ 0 & -1 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$ **7**

(i) Is A and B are invertible?
(ii) Is B right inverse of A?
(iii) Is A left inverse of B?

OR

A) Find the QR decomposition of the matrix $A = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$ **7**

B) If T:V→W is a linear operator, dim(V) = m and dim(W)= n then prove that r(T) ≤ min(m, n) also S: W→V is a linear operator, and V be finite dimensional then prove that r(ST) ≤ r(S). **6**

C) For T:V→W be linear operator then prove the following: **7**

(i) If T has an inverse S, the S is unique.
(ii) If T has left inverse, then T is one-one.
(iii) If T has right inverse R:W→V then T is onto, also T(x) = y if x = R(y).
(iv) T has inverse if and only if T is one-one and onto.

Q.5 A) Construct the Laguerre polynomials for v_1 (t) = t^i; i = 1,2,3 and $\langle x, y\rangle = \int_0^1 x(t)\overline{y(t)}\,dt.$ **7**

B) Find the orthogonal basis of the span of $(1,1,1)^T, (3,2,1)^T, (1,-3,-1)^T$. **6**

C) If M be a finite dimensional subspace of V. Then prove the projection of f on M exists and is unique for $f \in$ M. **7**

OR

A) Define Null space and Range of linear operator. Prove that the Null space and Range of linear operator are subspaces of norm space V and W respectively. **7**

B) State and prove triangular inequality for inner product space and discuss about the equality case. **6**

C) Let • be an inner product on a real vector space V. Then there is an orthonormal basis $(v_1, v_2, \ldots, v_n.)$ for V. If we represent vectors in coordinates with respect to this basis, say v = $[x_1, x_2, \ldots, x_n]^T$ and w=$[y_1, y_2, \ldots, y_n]^T then\ v.w = x_1y_1 + x_2y_2 + \cdots + x_ny_n$. **7**

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